Gravity in Undergraduate Thermal Physics Courses

Kartik Tiwari[1]

[1]Department of Physics, Ashoka University, Delhi-NCR 131029, India.

November 7, 2022

Abstract

A pedagogical aid is recommended in this paper to be included in undergraduate level thermal physics courses in order to introduce to students how inclusion of gravity challenges the conventional formulation of Laws of Thermodynamics. This should serve the purpose of stimulating further interest in the field of thermal physics by demonstrating its overlap with another important domain of physics i.e. Theory of Relativity.

1 Introduction

'If your theory is found to be against the second law of thermodynamics I can give you no hope; there is nothing for it but to collapse in deepest humiliation' - Arthur Eddington (1915)

From cosmology to information theory, thermal physics has witnessed some significant results in the last century on various fronts. An interesting set of results about thermodynamics and general relativity, however, remains inaccessible to undergraduate students due to the lack of necessary prerequisites. A pedagogical aid is suggested in this paper that can be easily incorporated in any undergraduate level thermal physics course after the introduction of the second law of thermodynamics.

Recent work by Santiago and Visser [2] reformulated the argument for the existence of Tolman-Ehrenfest effect in terms of special relativity. Their novel formulation has enabled a discussion of some important results that challenge the conventional forms of Laws of Thermodynamics on a significantly easier level (considering the original derivation employed relativistic hydrodynamics).

In this paper, I present three arguments that do not require a formal background in General Relativity and open up room for further investigation by the students. The first is a classical thermodynamics argument by Maxwell that disproves the existence of Temperature gradients at thermal equilibrium. The second argument by Santiago and Visser demonstrate a proof of Tolman Temperature gradients at thermal equilibrium while establishing that only gravity can create such gradients due to its universality. In the third argument, I argue that though only gravity can create temperature gradients, the presence of electric fields cannot be ignored because they too contribute to temperature gradients at thermal equilibrium via gravity itself.

2 Maxwell's Argument

Maxwell argued, on purely physical grounds, that temperature gradients cannot exist in bodies at thermal equilibrium [1].

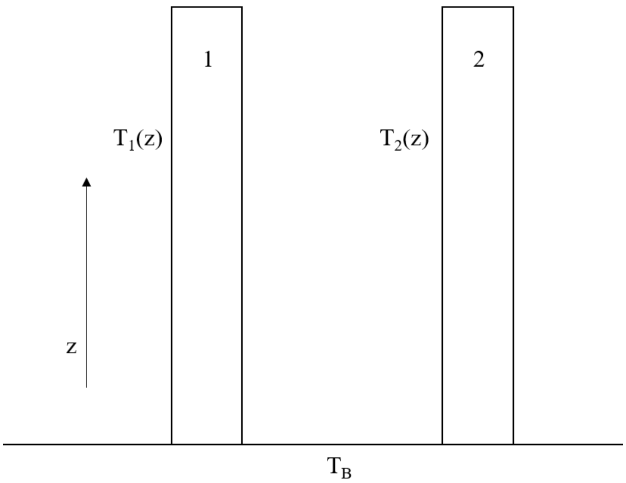

Figure 1: The hypothetical two column setting for Maxwell's argument

Imagine two columns standing on a thermally conducting surface. The base of the two columns are in thermal equilibrium with the conducting surface on which they stand (see Fig. 1). We start by assuming that, though the columns are in thermal equilibrium, they demonstrate a temperature gradient across the z-axis.

If at any given height z we come across a situation such that $T_1(z) > T_2(z)$ (or vice versa, since our system is completely arbitrary) then we can connect a conducting rod at that height. On this rod, heat flows from 1 towards 2. This additional heat that has been transferred to column 2 would be conducted, partly, towards the base $T_2(0)$, which heats up the base of 1 as well via the conducting surface over which the columns stand. When the base of column 1 gets heated, the heat flows upward in the column again forming a cycle (see Fig. 2).

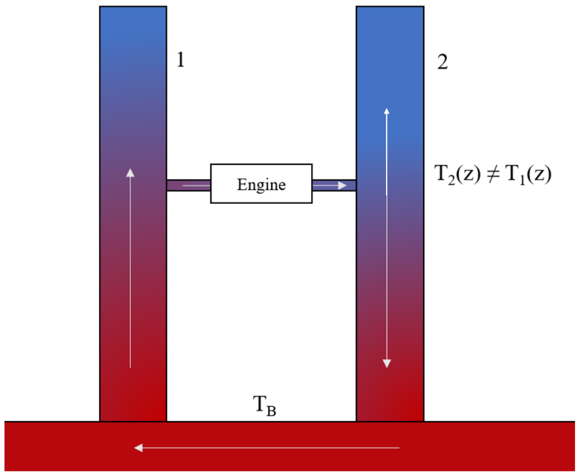

Figure 2: Inequality in temperature gradient creating a perpetual motion engine

We can place an engine on the conducting rod at height z. This renders us with a perpetual motion machine with the two columns perpetually acting as sources and sinks at different temperatures even at thermal equilibrium. This is an absurdity and clearly violates the second law of thermodynamics. Therefore, our assumption that $T_1(z) \neq T_2(z)$ must be false.

Maxwell follows the argument with an empirical observation and claims that since we do not see a temperature gradient in a column of ideal gas, no substance can demonstrate a temperature gradient at equilibrium. If such a gradient exists at all, it must be universal. Otherwise, it allows for the violation of Clausius's version of the second Law. Since the Clausius's and Kelvin's formulations are logically equivalent, students are encouraged to explain how the hypothetical apparatus would violate Kelvin's statement about the second law.

3 (Modified) Santiago-Visser's Argument

I will present a modified version of a simple proof of temperature gradients using a photon gas column as discussed in [2]. Tolman and Ehrenfest in their original work argued that temperature at thermal equilibrium is not constant but varies with spacetime curvature by employing General Relativistic Hydrodynamics, which is not something generally covered in the usual undergraduate physics track before a course in thermal physics. Santiago and Visser used only the idea of gravitational redshift to arrive at the same conclusion. Students are encouraged to look up the derivation for gravitational redshift as it can be understood using only concepts from special relativity discussed in an undergraduate electromagnetism course.

Unlike in the original Santiago-Visser paper, I would employ a spherically symmetric gravitational field instead of a constant gravitational field to streamline the transition into the third argument in this paper. Let us go through the proof now. Consider a spherically symmetric gravitational field around a massive body and a long photon gas column placed offset from radial direction as demonstrated in Fig. 3. Suppose an observer situated a large distance away from the photon gas column concludes that the spectral radiance of each section of the column demonstrate a peak - invariant of time - at the same wavelength. From Wein's Displacement Law, the observer infers that the body is in thermal equilibrium because the temperature of the column is spatially and temporally constant.

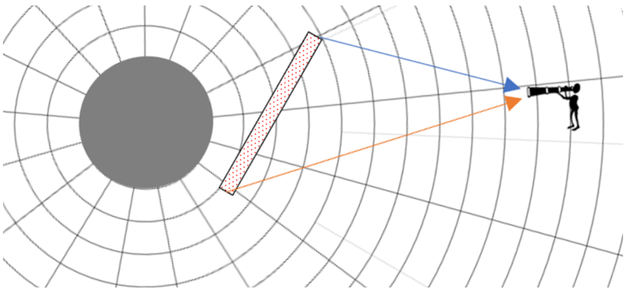

Figure 3: An observer observing the photons leaking from a photon gas column near a massive body

However, we know that a photon's motion through a gravitational well leads to energy-loss and, subsequently, a redshift in its wavelength. The expression to evaluate gravitational red-shift in a simple spherically symmetric gravitational field (or in other words, Schwarzchild geometry) is well understood to be -

$$\frac{\lambda_\infty}{\lambda_e} = \left(1 - \frac{R_s}{r}\right)^{-1/2}$$

where λ_e is the wavelength at emissions, λ_∞

is the wavelength observed at infinity, $R_s = 2GM/c^2$ is the Schwarzschild radius of the massive body and r is the radius at which the photon was initially emitted. For our observer situated large distance away from the body, $\lambda_o = \lambda_\infty$ is given by -

$$\lambda_o = \lambda_e \left(1 - \frac{2GM}{c^2 r}\right)^{-1/2}$$

From Wein's Displacement Law, we know $\lambda_{o_{max}} T_o = \lambda_{e_{max}} T_e$. Therefore, the temperature recorded by the observer would be off by a factor of -

$$T_e = \frac{T_o}{\sqrt{1 - \frac{2GM}{c^2 r}}}$$

In our hypothetical scenario, however, different sections of the photon column are clearly at different distances from the center of the massive body. This means one end of the column should experience a larger redshift than the other and this redshift should be reflected in the blackbody curve recorded by the observer. Thus, instead of seeing a constant spectral distribution throughout the column, the observer should be seeing displaced intensity peaks at different positions of the column. This is a clear contradiction. The only way the observer would observe spatially constant temperature throughout the column is if the there existed a temperature gradient that would exactly cancel out the gravitational redshift experienced by the photons. Therefore, we end up with an expression for locally measured temperature of the photon gas column that - though temporally constant - is a function of position -

$$T(r) = \frac{T_o}{\sqrt{1 - \frac{2GM}{c^2 r}}}$$

We were working in the Schwarzschild Geometry, the metric ($g_{\mu\nu}$) for which is

$$ds^2 = -\alpha c^2 dt^2 + \alpha^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

where,

$$\alpha(r) = \left(1 - \frac{R_s}{r}\right)$$

On comparing with our expression for locally measured temperature, it becomes clear that the Temperature gradient of the photon gas column follows the expression -

$$T(r) = \frac{T_o}{\sqrt{-g_{tt}(r)}} \qquad (1)$$

where T_o is constant as described earlier (g_{tt} is the component of the metric tensor that serves as the coefficient of $c^2 dt^2$ term). Notice that the gradient is independent of time. This is a counter-intuitive result because it implies that though there exists a temperature gradient in the photon gas column, the heat does not flow from hotter region to colder region. As is ubiquitous in measurements of length and time within framework of relativity, one now needs to be mindful of the distinction between 'coordinate' temperature and 'local' temperature (not standard terminology) as well.

3.1 Connecting the Two Pieces

Earlier we established using a Maxwellian argument that if there exist two unequal temperature gradients, we can run a perpetual motion engine. Then, we established that a photon gas column must have a temperature gradient for it to produce consistent results with observations. If we connect these two pieces of the argument, we can arrive at a general result about equilibrium temperature gradients i.e. Eq. 1 models the temperature gradient for all objects in static spacetimes.

The proof is completely analogous to the one discussed earlier. Let us start by assuming that only the photon gas column demonstrates a gradient at thermal equilibrium and another column made of some other material (say, ideal gas) does not. If we keep them parallel and very close to each other (Fig. 4) and connect them via conducting surface at the base then we arrive at the exact same scenario mentioned in Fig. 2. Therefore, on connecting a conducting rod between the two columns, we can extract work from heat transfer indefinitely and violate second law.

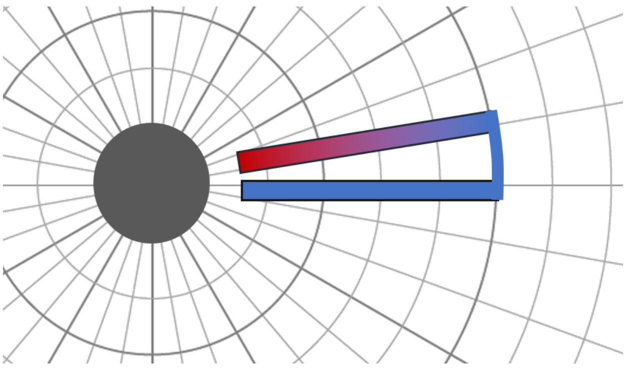

Figure 4: An observer observing the photons leaking from a photon gas column near a massive body

3.2 Universality of Gravity

To avoid perpetual motion, we must accept that all temperature gradients at thermal equilibrium in a specific geometry must follow the same relation described by Eq. 1 (this expression, however, holds true only for static spacetimes). The temperature gradient is caused due to spacetime curvature around the massive body and, therefore, all massive particles and radiation interacting with the spacetime curvature should experience the same temperature gradient. In the non-relativistic limit $c \to \infty$ the gradient would be indeed in the limit $\nabla T(r) \to 0$. This establishes the *Universality of Gravity*.

Having established the universality of gravity and the temperature gradient associated with gravity at thermal equilibrium, we move our attention to the possibility of temperature gradients formed by electromagnetic field. The following is also a part of [2] and is developed over Maxwell's two column argument. We consider a similar apparatus as described earlier with few minor adjustments. Suppose one of the columns is filled with very low density electron gas and the entire apparatus is subjected to an Electric Field $\vec{E}$ as in Fig. 5. Does $\vec{E}$ produce a temperature gradient at thermal equilibrium? Let's start by assuming it does. What happens next?

If there is a temperature gradient produced due to the electric field then it must only affect those particles that interact with $\vec{E}$ to have any causal relationship in the first place. If the adjacent column is made of non-

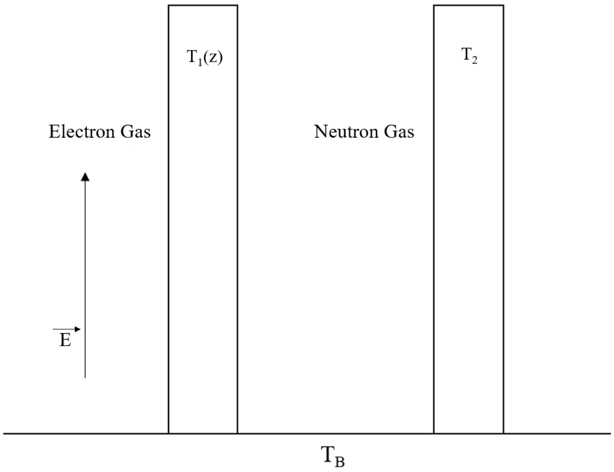

Figure 5: Electron Gas and Neutron Gas Columns Exposed to $\vec{E}$

interacting particles (such as Neutron Gas) then $\vec{E}$ has no causal influence over the second column. Thus, we are in a situation where there exists a temperature gradient in the electron gas column and no temperature gradient in the neutron gas column at thermal equilibrium. After having repeated the Maxwell's argument twice, this should be immediately alarming. At the risk of being redundant, let me explicitly state the problem again. If at any height in the system at thermal equilibrium, the temperature of the two columns is in in equal, we can run an engine between them using the thermal gradient *ad infinitum*.

This result can be further generalized. Any force for which there exists particles that do not interact with the force cannot produce a temperature gradient at thermal equilibrium. Gravity is universal and, hence, causes Tolman-Ehrenfest Effect at thermal equilibrium. Electric force is not universal and, hence, thermal equilibrium is unaffected by it.

4 Influence of Electric Fields on Temperature Gradients

Notice that gravity was ignored while probing effects of electric field. Let us factor effects of gravity again. Does presence of electric field affect the temperature gradient at equilibrium now? Santiago and Visser established electric fields should have no contribution in the development of temperature gradient. But let us recall two of (classical) Maxwell's equations -

$$\nabla.\vec{E} = \frac{\rho}{\epsilon_0} \text{ and } \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Clearly, to have an electric field, one either needs a charge density or a changing magnetic field. In other words, electric field lines either originate/end on charges or form closed loops.

4.1 Reissner–Nordström Geometry

In general relativity, anything with Energy-Momentum affects the geometry of spacetime. The simplest analysis of gravitational influence of massive charged objects is **Reissner–Nordström** metric, which is a static spacetime. The metric that describes the spacetime near a charged spherically symmetric blackhole is

$$ds^2 = -\Delta c^2 dt^2 + \Delta^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

where the coefficient Δ is

$$\Delta(r) = \left(1 - \frac{R_s}{r} + \frac{R_Q^2}{r^2}\right)$$

Like earlier, $R_s = 2GM/c^2$ is the Schwarzschild radius and $R_Q = (Q^2G)/(4\pi\epsilon_0 c^4)$ is a characteristic length defined by the net charge content of the body. Clearly, when we set $Q = 0$, we simply get a Schwarzschild geometry. If the charged blackhole is also spinning then we call it a Kerr-Newman geometry. Since we are not looking at stationary spacetimes and magnetic mono-poles do not exist, we will not worry much about the magnetic component for now. For emphasis, I reiterate the key takeaway - presence of charge affects the geometry of spacetime.

It is instructive to think of the massive bodies as blackholes because of the exaggerated gravitational effect they cause. Otherwise, the argument is completely general and works around any charged massive body with spherical symmetry.

4.2 Are temperature gradients affected by $\vec{E}$?

We now return back to the original question - are temperature gradients at equilibrium (in static spacetime) affected by the presence of electric field? If the electric field could influence the temperature gradient at thermal equilibrium then it would allow for the existence of perpetual motion machines. Therefore, we rule out the possibility of a contribution by the electric field at thermal equilibrium. However, in a static spacetime, electric fields cannot exist without a charge. Since the existence of charge affects the spacetime metric and the spacetime metric affects the temperature gradient, the presence of $\vec{E}$ does indeed contribute to the temperature gradient in an indirect manner.

Obviously, in routine thermodynamic experiments, ∇T is incredibly small (with or without the presence of charge). For all practical purposes, we ignore the Tolman gradient but it must exist to ensure the logical consistency of our theories.

5 Conclusion

A pedagogical aid was suggested in this paper to introduce early undergraduate students to the limitations of conventional ways of expressing the laws of thermodynamics. The work presented in this paper also included an elementary clarification regarding the nature of causal relationship between thermal equilibrium temperature gradients and presence of electric field. On studying the presence of electric field carefully and keeping universality of gravity in mind, it was established that electric fields influence temperature gradients even though electromagnetism is not universal like gravity. Since *this influence is carried out via gravity itself*, the conclusion remains consistent with the well established principle - Gravity is the only force capable of producing temperature gradients at equilibrium.

Even though the results of these

thought experiments may seem inconsequential due to their negligible scales, the existence of these temperature gradients at thermal equilibrium requires a reformulation of other laws of thermodynamics. The definition of Temperature given by Zeroth Law is not consistent with relativity. The directional flow of heat prescribed by Second Law is not persevered due to the existence of stable gradients. Fortunately, the field of relativistic thermodynamics is well established and such issues have already been resolved.

6 Discussion

A 45 minute talk based on this paper was given to a general physics audience and received a positive response in terms of stimulating interest in relativistic thermodynamics and exposing the audience to novel ideas not discussed in their thermal physics courses. However, as an effective way to test the comprehensibility of these concepts at an undergraduate level, a small quiz can be designed that students answer after an hour-long lecture on the aforementioned topics. Such a data driven analysis, though relevant and helpful in testing new pedagogical tools in general, is beyond the ambit of this specific paper.

An effective teaching module should open up the possibility of, what I call, a forward and a backwards approach to self learning. A forward approach takes the results taught in the classroom as granted and applies the newly acquired theoretical toolkit to further problems in physics. This may include reformulating certain standard textbook problems in terms of temperature gradients or exploring relativistic thermodynamics and its applications in cosmology in greater detail. On the other hand, a backward approach questions the premises taken as true during the classwork and tries to reach a more fundamental explanation for the unproved results. This might include studying about Einstein's field equations to understand how and why stress energy tensor affects gravity, understanding the more general temperature gradient results for stationary spacetime, etc. The contents of this paper open up both these channels for the interested students to engage with the subject further.

Acknowledgments

The author would like to thank Prof. Vikram Vyas who suggested that the question was worth an inquiry, Naxxatra for organizing a public lecture based on the contents of this paper and Vipasha Barot for her constant support.

References

[1] Maxwell, James Clerk (1868) The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science 35

[2] Santiago, Jessica and Matt Visser (2018) Physical Review D 98

[3] Santiago, Jessica and Matt Visser (2018) The European Journal of Physics 40

[4] Tolman, Richard C and Paul Ehrenfest (1930) Physical Review 36